Anomaly Correction by Optimal Trading Frequency

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Introduction

- Malkiel (1995), price follows random walk.
- De Bondt and Thaler (1984), provided empirical evidence to show that the overreaction hypothesis is consistent in market level. That is, they have shown that portfolios of prior "losers" are found to outperform prior "winners".
- Question: how to consistently buy low (or sell high)? i.e. How to get rid of the volatility and "clean" this market so that the price chart looks more like the GDP chart?

Figure 1

Figure: The return after interest rate with $1 investment in the market from July of 1926 to June of 2016. *Source: From Ken French Data Library.*

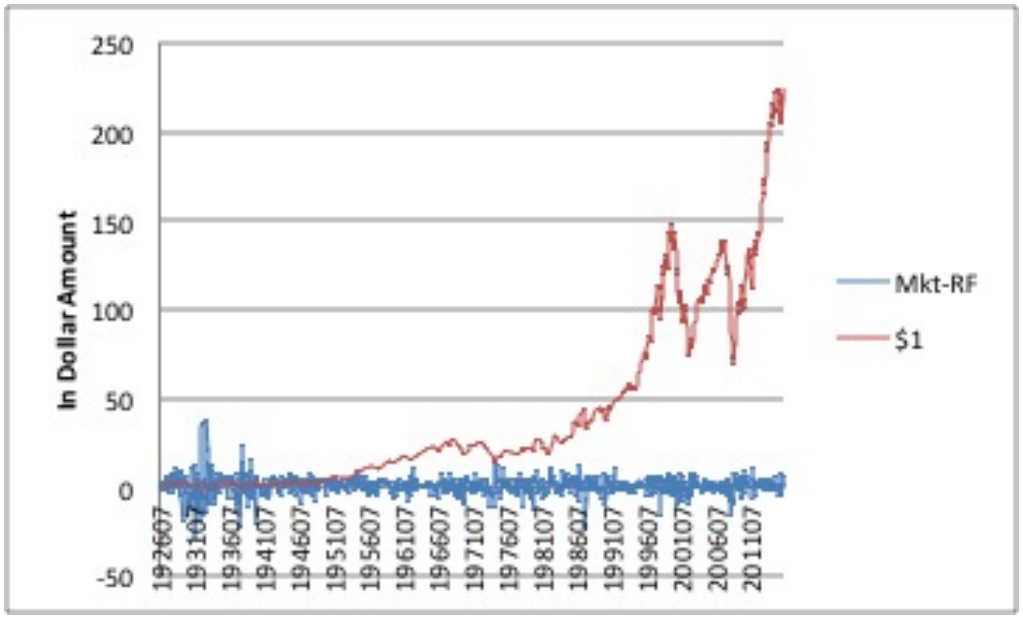
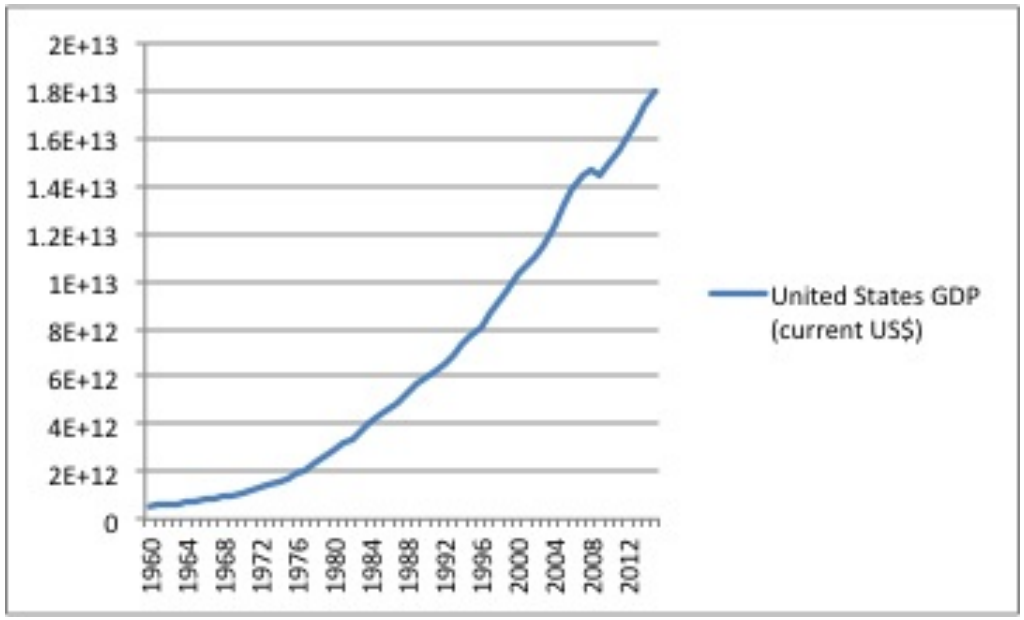


Figure 2

Figure: The GDP in US Dollars from 1960 to 2015. *Source: From World Bank Library Current GDP (in Dollars) for United States.*



Moving Average

Definition

Simple moving average (SMA) takes the average of prices traded n time period(s) in the past,

$$SMA_n = \frac{1}{n} \sum_{t=0}^{n} P_t$$

.

Price-to-Moving Average

Definition

Price-to-Moving Average is the ratio of price over a selected period of moving averages. For n periods, we have

$$P_t - SMA_n = \frac{P_t}{\frac{1}{n}\sum_{t=0}^{n} P_t} - 1$$

.

Trading Frequency

Definition

The particular event k is the product of δ and total observations with

$$\exists h \in \mathbb{R}, \delta = Pr(\sigma_{t,n} \geq h\sigma_{t,n}) = \frac{\sum_{(t,n)\to(t',n')}^{(t'',n'')}(h\sigma_{t,n})}{\sum_{(t,n)} \sigma_{t,n}}$$

while

$$\sigma_{t,n} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}((p_t - SMA_i) - \frac{1}{n}\sum_{i=1}^{n}(p_t - SMA_i))^2}$$

Optimal Trading Frequency

The first goal is to find the optimal price-to-moving averages level. We need to take the first order derivative of price-to-moving averages and set the result to zero. We have the following theorem. (Proof see paper)

Theorem

$\forall n$, let $\triangle$ be a small unit of time period, when $\sum_{t=0}^{n} P_t = \sum_{t=0}^{n} P_{t+\triangle}$, we have a critical price to act on it.

Interpretation

* The interpretation is fairly intuitive.
* We look at a series of price and the price goes up or down.
* Then we have a series of average prices. We look at the ratio of price over moving averages. We calculate the sum of prices in the past certain of time nodes.
* The optimal price level occurs at a time when the sum of the averages at two time nodes does not change at all (or change tiny little).
* For example, we calculate the sum of hundreds of prices in the past. If the price keeps going down, we are supposed to get a sum smaller and smaller, yet one day the sum stays the same all of the sudden. Then this price can be critical to buy and vice versa.

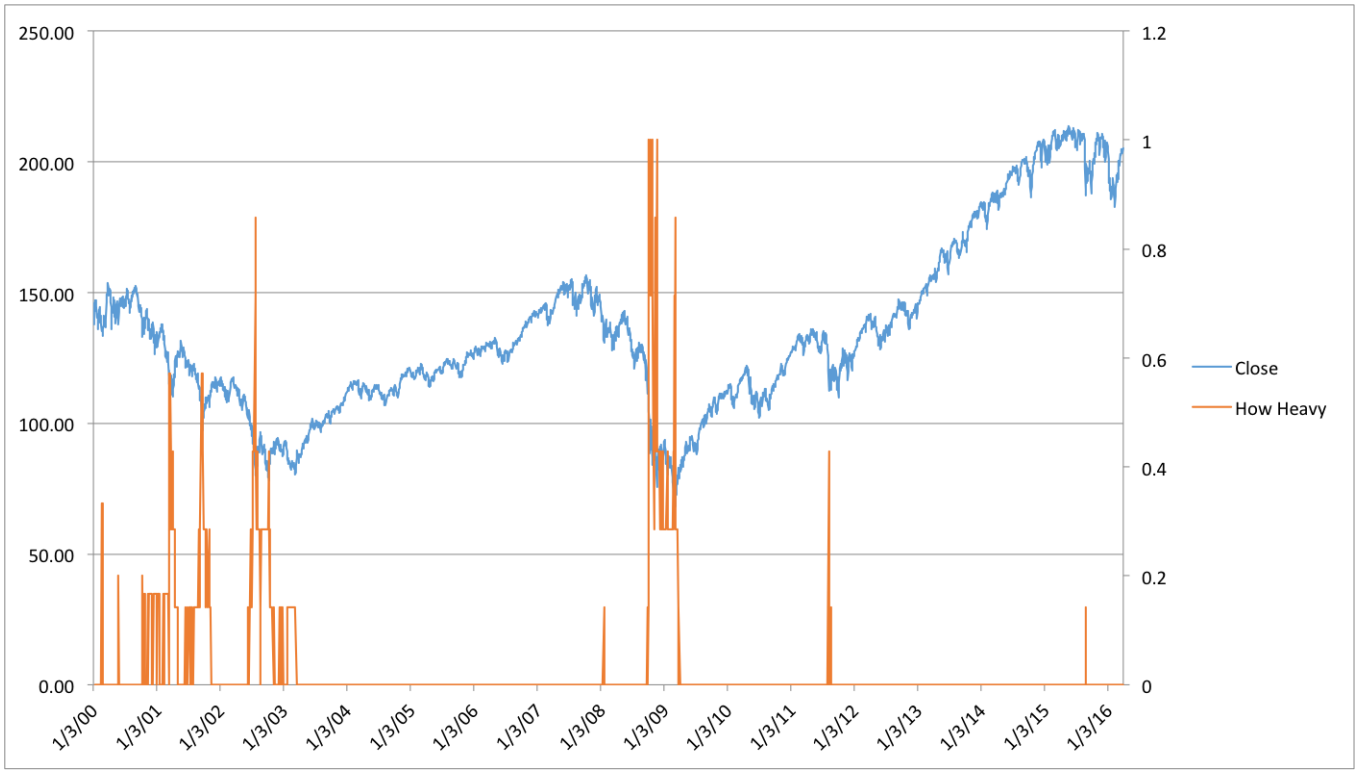
Summary

* In this paper we present a method to pick a critical price by using moving averages and investor trading frequency.
* Assume random walk and sufficient liquidities.
* Goal: getting rid of the ups and downs of the equity market, hence less volatile.

Discussion

Figure: The graph shows S&P 500 from January 2000 to January 2016. The model takes 10-, 20-, 30-, 50-, 100-, 200-, 300-day to be moving average periods and calculate the price-to-moving average ratio by investor trading frequency to be less than 10%. The spikes from x-axis are buy signal calculated based on critical price level. The higher the spikes are the heavier an investor should buy. *Source: From Yahoo Finance Library.*

Discussion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How Heavy

Potential Research Question

* This paper touched slightly on higher level thinking in binomial theory, but it is not complete yet.
* We can add in machine learning to improve our analytic ability on trading frequency.
* Or even AI?